



# Linear Algebra

## Course booklet

*Linear Algebra for college & university students. Contains vector calculus/spaces, matrices and matrix calculus, inner product spaces, and much more.*



# ABOUT & PRICING

## About Bolster Academy

Bolster Academy offers:

- a homework, practice and **learning environment**;
- personalised **feedback** on all answer attempts;
- different **testing and assessment** tools;
- customisable **mathematics courses** with explanations, examples, and endless **randomised practice exercises**;
- an authoring tool to **create original material**;
- **learning analytics** giving detailed insight into student performance;
- **integration** with your LMS/VLE.

Our learning environment guides students along as they solve problems. When doing exercises, students can enter open answer calculations or mathematical formulas. The software will analyse their answer and provide targeted feedback and hints helping the student understand the next step in the solution process, and/or highlight any mistakes they made.

***Bolster Academy increases student engagement and saves teachers time checking and grading!***

## Pricing

Bolster Academy partners with higher education institutions on a SAAS licensing basis.

The cost for the platform starts at € 5.50 per student per year, with an additional per student per year fee of € 7.50 per course.

A second licensing model is one in which students pay for their own license in our webshop.

Our digital courses are a fully interactive alternative for paper books and offer a personalised and adaptive learning experience that fits today's generation of students.

## How are courses structured?

The courses are structured in chapters and subchapters consisting of units. Every chapter starts with an introduction and ends with a conclusion. The unit subjects are listed in more detail on the following pages.

Each unit consists of (at least) one theory page and one package of exercises.

**Theory pages** contain explanations, (randomized) examples and visualisations and (interactive) graphs.

The packages of **exercises** contain on average around 10 exercises. Each of these exercises are randomised, allowing for endless practicing, and include targeted hints and personalised feedback for the students while solving the exercises.

# COURSE CONTENT

## Chapter 1: Complex numbers (17 topics)

1. *Introduction to complex numbers (4 topics)*
  - a. Imaginary numbers
  - b. The notion of complex numbers
  - c. Polar coordinates
  - d. Real and imaginary part
2. *Calculating with complex numbers (4 topics)*
  - a. Calculating with polar coordinates
  - b. The quotient
  - c. Complex conjugate
  - d. Geometric interpretation
3. *Complex functions (4 topics)*
  - a. Complex exponents
  - b. Rules of calculation for complex powers
  - c. Complex sine and cosine
  - d. Complex logarithm
4. *Complex polynomials (5 topics)*
  - a. The notion of a complex polynomial
  - b. Factorization of complex polynomials
  - c. Zeros of complex polynomials
  - d. Fundamental theorem of algebra
  - e. Real polynomials

## Chapter 2: Vector calculus in plane and space (21 topics)

5. *Vectors in planes and space (4 topics)*
  - a. The notion of vector
  - b. Scalar multiplication
  - c. Addition of vectors
  - d. Linear combinations of vectors

6. *Straight lines and planes (2 topics)*
  - a. Straight lines and planes
  - b. Parametrization of a plane
7. *Bases, coordinates and equations (5 topics)*
  - a. The notion of base
  - b. Coordinate space
  - c. Straight lines in the plane coordinates
  - d. Planes in coordinate space
  - e. Lines in the coordinate space
8. *Distances, angles and dot product (5 topics)*
  - a. Distances, Angles and dot products
  - b. Dot product
  - c. Properties of the dot product
  - d. The standard dot product
  - e. Normal vectors
9. *The cross product (5 topics)*
  - a. Cross product in 3 dimensions
  - b. The concept of volume in space
  - c. The volume of a parallelepiped
  - d. Properties of cross product
  - e. The standard cross product

### **Chapter 3: Systems of linear equations and matrices (21 topics)**

10. *Linear equations (4 topics)*
  - a. The notion of linear equation
  - b. Reduction to a base form
  - c. Solving a linear equation with a single unknown
  - d. Solving a linear equation with several unknowns
11. *Systems of linear equations (5 topics)*
  - a. The notion of a system of linear equations
  - b. Homogeneous and inhomogeneous systems
  - c. Lines in the plane

- d. Planes in space
- e. Elementary operations on systems of linear equations

### 12. *Systems and matrices (7 topics)*

- a. From systems to matrices
- b. Equations and matrices
- c. Echelon form and reduced echelon form
- d. Row reduction of a matrix
- e. Solving linear equations by Gaussian elimination
- f. Solvability of systems of linear equations
- g. Systems with a parameter

### 13. *Matrices (5 topics)*

- a. The notion of a matrix
- b. Simple matrix operations
- c. Multiplication of matrices
- d. Matrix equations
- e. The inverse of a matrix

## Chapter 4: Vector spaces (14 topics)

### 14. *Vector spaces and linear subspaces (4 topics)*

- a. The notion of vector space
- b. The notion of linear subspace
- c. Lines and planes
- d. Affine subspaces

### 15. *Spans (5 topics)*

- a. Spanning sets
- b. Operations with spanning vectors
- c. Independence
- d. Basis and dimension
- e. Finding bases

### 16. *More about subspaces (2 topics)*

- a. Intersection and sum of linear subspaces
- b. Direct sum of two linear subspaces

### 17. *Coordinates (3 topics)*

- a. The notion of coordinates
- b. Coordinates of sums of scalar multiples
- c. Basis and echelon form

## Chapter 5: Inner product spaces (14 topics)

### 18. *Inner product, length, and angle (3 topics)*

- a. The notion of inner product
- b. Angle
- c. Perpendicularity

### 19. *Orthogonal systems (3 topics)*

- a. The notion of orthonormal system
- b. Properties of orthonormal systems
- c. Constructing orthonormal bases

### 20. *Orthogonal projections (3 topics)*

- a. Orthogonal projection
- b. Orthogonal complement
- c. Gram-Schmidt in matrix form

### 21. *Complex inner product spaces (5 topics)*

- a. Inner product on complex vector spaces
- b. Orthonormal systems in complex vector spaces
- c. Orthogonal complements in complex inner product spaces
- d. Complex orthogonal complements
- e. Gram-Schmidt in complex inner product spaces

## Chapter 6: Linear maps (19 topics)

### 22. *Linear maps (9 topics)*

- a. The notion of linear map
- b. Linear maps determined by matrices
- c. Composition of linear maps
- d. Sums and multiples of linear maps
- e. The inverse of a linear map

- f. Kernel and image of a linear transformation
- g. Recording linear maps
- h. Rank-nullity theorem for linear maps
- i. invertibility criteria for linear maps

### **23. Matrices of linear maps (7 topics)**

- a. The matrix of a linear map in coordinate space
- b. Determining the matrix in coordinate space
- c. Coordinates
- d. Basis transition
- e. The matrix of a linear map
- f. Coordinate transformations
- g. Relationship to systems of linear equations

### **24. Dual vector spaces (3 topics)**

- a. The notion of dual space
- b. Dual basis
- c. Dual map

## **Chapter 7: Matrix calculus (16 topics)**

### **25. Rank and inverse of a matrix (2 topics)**

- a. Rank and column space of a matrix
- b. Invertibility and rank

### **26. Determinants (7 topics)**

- a. 2-dimensional determinants
- b. Permutations
- c. Higher-dimensional determinants
- d. More properties of determinants
- e. Row and column expansion
- f. Row and column reduction
- g. Cramer's rule

### **27. Matrices and coordinate transforms (4 topics)**

- a. Characteristic polynomial of a matrix
- b. Conjugate matrices
- c. Characteristic polynomial of a linear map



d. Matrix equivalence

### **28. Minimal polynomial (3 topics)**

- a. Cayley-Hamilton
- b. Division with remainder for polynomials
- c. Minimal polynomial

## **Chapter 8: Invariant subspaces of linear maps (14 topics)**

### **29. Eigenvalues and eigenvectors (3 topics)**

- a. Diagonal form
- b. Eigenspace
- c. Determining eigenvalues and eigenvectors

### **30. Diagonalizability (4 topics)**

- a. The notion of diagonalizability
- b. Diagonalizability and minimal polynomial
- c. The greatest common divisor of two polynomials
- d. The Euclidean algorithm

### **31. Invariant subspaces (7 topics)**

- a. The notion of an invariant subspace
- b. The extended Euclidean algorithm
- c. Direct sum decomposition into invariant subspaces
- d. Generalized eigenspace
- e. Jordan normal form
- f. From real to complex vector spaces and back
- g. Real Jordan normal form for non-real eigenvalues

## **Chapter 9: Orthogonal and symmetric maps (17 topics)**

### **32. Orthogonal maps (5 topics)**

- a. The notion of orthogonal map
- b. Properties of orthogonal maps
- c. More properties of orthogonal maps
- d. Orthogonal matrices
- e. Orthogonal transformation matrices

### ***33. Classification of orthogonal maps (3 topics)***

- a. Low-dimensional orthogonal maps
- b. Jordan normal form for orthogonal maps
- c. Classification of orthogonal maps

### ***34. Unitary maps (2 topics)***

- a. The notion of unitary map
- b. Diagonal form for unitary maps

### ***35. Isometries (3 topics)***

- a. The notion of isometry
- b. Equivalence of isometries
- c. Characterisation of isometries

### ***36. Symmetric maps (4 topics)***

- a. The notion of symmetric map
- b. Connection with symmetric matrices
- c. Properties of symmetric maps
- d. Orthonormal bases and symmetric maps

### ***37. Application of symmetric maps (4 topics)***

- a. Quadratic forms
- b. Quadrics
- c. Least square solutions of linear equations
- d. Singular value decomposition

## **Chapter 10: Differential equations and Laplace transform (9 topics)**

### ***38. Differential equations and Laplace transform (9 topics)***

- a. The Laplace transform
- b. The inverse Laplace transform
- c. Laplace transforms of differential equations
- d. Convolution
- e. Laplace transforms of Heaviside functions
- f. Laplace transforms of periodic functions
- g. Riemann-Stieltjes integration
- h. Laplace transforms for delta function
- i. Transfer and response functions



**bolster.academy**  
**info@bolster.academy**  
**+3120 752 0000**

**Science Park 402**  
**1098XH Amsterdam**  
**The Netherlands**

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