SOWISO

Part 2 of a four-part series

Complex problems and solutions in maths education

Motivate students for maths learning

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1. Introduction

The study of mathematics is foundational for plenty of other subjects. Engineering, Biology, Nursing, Business and Chemistry are all subjects which require a fundamental understanding of mathematics when studied at degree level. Many universities require their non-mathematics students to pass a foundational mathematics course before they can continue with their proper degree. These foundational maths courses can pose significant problems for teachers and students.

Foundational mathematics programmes are often large and include students from different disciplines. Students have varied levels of prior maths knowledge, differing levels of motivation and interest and frequently, different levels of spoken language. International students may have followed very different mathematics curricula to the local one. Bamforth et al. note, 'students without the assumed mathematics knowledge and skill may be at risk of failing or underachieving'. Sadly a failing grade in a foundational mathematics course can result in interrupted main study, repeated classes and ultimately failing a degree.

Teachers commonly struggle with large classes and a resulting lack of time and focus; apathy from students and high failure rates for an important course.

These unique problems have led to broader fears about both the quality of mathematics education at universities and HE institutions and the abilities of students to study mathematics. This caused the London Mathematical Society to cite 'unprecedented concern' over the mathematical preparedness of undergraduates. While this is an issue common to maths-based degrees as well as degree courses including a foundational maths course, the so-called 'Maths Problem' is gaining notoriety.

Various solutions have been offered for this well documented problem including intensive summer schools or crash courses, online instructional materials, on-site support centres, computer aided assessments and problem based learning strategies. These solutions have had differing degrees of success across various institutions, and are often successful only with certain students or within certain time frames. SOWISO, on the other hand, has taken a different approach.

[1] *Broadbridge. P. & Henderson, S. (2008) 'Mathematics Education for 21st Century Engineering Students: Final Report', Australian Mathematical Sciences Institute & Carrick Institute. Strawberry HIlls, NSW: Carrick Institute;*

[2] *Bialek, W. & Botstein, D. (2004) 'Introductory science and mathematics education for 21st-century biologists', Science, 303(5659): 788-790.*

[3] *Croft, A.C., Harrison, M.C. & Robinson, C.L. (2009) 'Recruitment and retention of students - an integrated and holistic vision of mathematics support', International Journal of Mathematical Education in Science and Technology, 40(1): 109-125.*

[4] *Loughlin, Wendy A., Watters, Dianne J., Brown, Christopher L. & Johnston, Peter R. (2015) 'Snapshot of mathematical background demographics of a broad cohort or first year chemistry science students', International Journal of Innovation in Science and Mathematics Education 23(1): 21-36.*

[5] *Bamforth, S., Robinson, C.L., Croft, T. & Crawford, A. (2007) 'Retention and progression of engineering students with diverse mathematical backgrounds', Teaching Mathematics and its applications, 26(4): 156-166.*

[6] *London Mathematical Society (1995) Tackling the mathematics problem. London: London Mathematical Society. Available at: http://mei.org.uk/files/pdf/Tackling_the_Mathematics_Problem.pdf*

[7] *Bamforth et al (2007)*

[8] *Llorens, M., Nevin, E. & Mageen, E. et al. (2014) 'Online resource platform for mathematics education', in Proceedings 44th Annual Frontiers in Education (FIE) Conference (22-25 October) Madrid: 1865-1872.*

[9] *Broadbridge & Henderson (2008)*

[10] *M. Cazzola, M. (2008), "Problem-Based Learning and Mathematics: Possible Synergical Actions", in L. Gomez Chova, D. M. Belenguer, and I. Candel Torres (eds.), ICERI2008 Proceeding, IATED (International Association of Technology, Education and Development), Valencia, Spain, 2008.*

[11] *For a more detailed analysis of various solutions to this problem and a discussion of their impact, see the SOWISO Whitepaper, Best practices: Implementing personalised maths e-learning tools in undergraduate courses www.sowiso.nl/en/*

[12] *For example Bamforth et al. (2017) have shown clearly that much maths support tapers off after the first year, leading to a drop in results by second year.*

2. The digital solution

SOWISO has spent a long time considering and researching the issue of mathematics learning for non-maths students and the unique issues it involves. Any solution to the maths problem needs to be efficient, effective and relatively low-cost, in order to be widely adopted by the HE institutions who need it. SOWISO has combined two of the most effective strategies to challenge this issue - those of digital learning[13] and problem based learning[14, 15] approach.

This has resulted in the SOWISO online personalized learning platform, which is specifically designed to assist students and teachers operating in maths-related subjects.

Digital learning platforms are becoming increasingly popular solutions across all educational sectors. Although online learning platforms are in no way a substitute for properly trained and funded teaching staff, they can provide significant assistance for both students and teachers. The scope of such tools is vast, as is the variety of educational tools and their uses.

Digital tools are proven to be particularly effective when part of a blended learning approach.[16, 17] This approach combines face-to-face teaching with computer technologies, allowing both digital technologies and teaching staff to 'fill in the gaps' left by the other. The combination, if implemented correctly, can:

- Allow teaching staff more contact time with students
- Provide options for students with different working and learning patterns
- Lower institutional costs

A particularly popular form of blended learning is colloquially known as 'Flipping the Classroom[18]', wherein students complete the majority of their learning work at home, usually with the assistance of a digital learning platform. Teacher-facing time is then used for problem solving, questions and student-teacher discussions.

However, many teachers and administrators shy away from such online platforms and digital tools because of fears about the complexity of the tool or worries about the difficulties of integration[19] into already existing technologies. Often teachers feel under-confident about new technology and prefer to rely on traditional pedagogies and teaching tools.

Many studies have demonstrated that if online platforms are integrated effectively, and teachers are confident with using new tools, then they create significant benefits for students, teachers and administrative staff in higher education.

Various online platforms are used across Higher Education, with differing results and for different purposes. The SOWISO platform is based on giving mathematics students repeated chances to learn and practice the material, improving their retention and understanding with each use.

SOWISO uses repetitive formative and diagnostic assessment to allow students repeated opportunities to learn and practice mathematical concepts.[20] Teachers are provided in-depth and accurate overviews of their students' results, allowing them to determine early on which students are struggling or which concepts are difficult for many to grasp. SOWISO improves teaching and learning from both ends of the spectrum, improving the experience for all.

Benefits of the SOWISO platform include:

- Full control over learning material for teachers, including the ability to edit or reorder pre-existing material or create learning materials from scratch
- Randomized and open questions
- Adaptive learning functionality, which chooses the difficulty level of the next question based on previous answers
- Ability of students to access learning materials at any time
- Advanced learning analytics for teachers, based on a variety of criteria
- Immediate and intelligent feedback for maths problems completed
- Formative and non-formative assessments
- Various degrees of integration with Learning Management Systems, including Canvas, Blackboard and Moodle

SOWISO technology has been subject to rigorous academic assessment and several studies on its successes have been written.[21] Part of creating a useful tool involves delving deeply into the nature of the problems faced by teachers and students, both before and during use of the SOWISO tool. This following section will highlight this more clearly, before moving onto a detailed analysis of one specific problem from the perspective of teaching staff.

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[13] Llobregat-Gomez et al. (2015) cite a study which yielded an attendance rate of 95% with a 90% success rate for students, a number previously unheard of in the course.

[14] *Şendağ, S. & Odabaşı, H. F., 'Effects of an online problem based learning course on content knowledge acquisition and critical thinking skills', Computers & Education 53(1): 132-142.*

[15] *Lazakidou, Georgia & Retalis, Symeon (2010) 'Using computer supported collaborative learning strategies for helping students acquire self-regulated problem-solving skills in mathematics', Computers & Education 54: 3-13.*

[16] *Llobregat-Gomez et al (2015)*

[17] Ndloui, Mdutshekelwa C. & Mostert, I. (2018) 'Teacher Perceptions of Moodle and Throughput in a Blended Learning Programme for In-Service Secondary School Mathematics Teachers', Africa Education Review, 15(2): 131-151.

[18] Educational Horizons (2011) 'Flipping the classroom', Educational Horizons 90(1): 5-7.

[19] Garrison, D. R. & T. Anderson (2003) E-learning in the 21st Century: A framework for research and practice. London: Routledge/Falmer.

[20] Heck, A. (2017) 'Using SOWISO to realize interactive mathematical documents for learning, practising, and assessing mathematics', MSOR Connections 15(2).

[21] See Heck, A. & Brouwer, N. (2015) "Digital assessment-driven examples-based mathematics for computer science students", in N.Amado & Carreira (eds.) Proceedings of the 12th International Conference on Technology in Mathematics Teaching. University of Algarve, Portugal, pp.403-411.

3. Purpose of this paper

Over the years SOWISO has gleaned much experience of employing digital solutions to assist mathematics learning in Higher Education. Over this period we have developed an insight into some of the best practices for integration and use of such tools. Although digital learning platforms take much inspiration from traditional pedagogies, sometimes teaching staff feel adrift or unsure when using them in a classroom context. This paper forms part of a wider series which considers the issues that teaching staff regularly face while teaching maths, and digs into these problems more deeply.

We hope that by considering a variety of issues in more depth, as well as considering some solutions that exist, we can assist teachers in using digital platforms in their work. Digital platforms can allow teaching staff to automate certain processes, provide a variety of helpful data and generally allow them to spend more time on the intricacies of teaching, where they can be of the most help to their students.

This paper will consider two intricately linked subthemes which link to the goal of improving student engagement. While the subthemes of motivation and support are linked, they require significant attention. For this reason, the whitepaper has been split into two parts - Part 1 covering motivation and Part 2 covering supporting students and learning analytics. You will find links to the other paper on the final page.

4. Engagement in Maths learning

Academic engagement - what it is, how to get it, how to prove or improve it - is one of the most studied concepts in education. Every teacher and educational institution wants to engage their students. Engagement leads to better outcomes in learning, as well as to other significant benefits. While engagement is a complicated concept to define, it is generally seen as '[a]ctive involvement, commitment, and attention as opposed to apathy and lack of interest[22,23]'. Other academics have added that engagement involves student investment in a subject, a willingness to be involved in learning, active participation and seeing the value of a subject in the wider world[24].

Engagement is particularly important in the field of mathematics - a subject known to 'turn off' students resulting in a variety of long term consequences, from individual student futures to the economic development of a nation[25]. A variety of factors influence student success in maths[26], but engagement is a crucial one, and one that teachers can have significant impact on.

While the topic of engagement is complex, this paper will focus on two aspects of engagement which can be influenced by teachers - motivation and support. Motivating students, while simultaneously supporting them (particularly those students who struggle with mathematical learning) are two crucial teaching activities which address and assist the full spectrum of students.

Engaging students through motivation as well as supporting them in their struggles is an intrinsic part of teaching. Many teachers already perform this role admirably, but can find doing this in a digital environment bewildering. While digital platforms show clear benefit by allowing teaching staff to automate repetitive tasks with ease, and administer large and diverse classrooms smoothly, the tasks of motivation and support are less obviously achievable. This paper will explore strategies for motivation and support within digital learning platforms.

If teaching staff can harness digital platforms such as SOWISO for these tasks, there are huge benefits to be found. In using digital platforms to simultaneously motivate and support large classrooms, teachers can improve outcomes for mathematics learning across HE institutions. The scalability of platforms like

SOWISO can maximise the benefits for large, diverse classes filled with students dealing with various degrees of apathy or disengagement. This ultimately helps to tackle the Maths Problem in an integrated and helpful way.

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[22] *Singh, Kusum, Granville, Monique & Dika, Sandra (2002) 'Mathematics and Science Achievement: Effects of Motivation, Interest, and Academic Engagement', The Journal of Educational Research 95(6): 323-332. pp.324.*

[23] *Newmann, F. M., Wehlage, G. G., & Lamborn. S. D. (1992) "The significance and sources of student engagement", in F. M. Newmann (ed.), Student engagement and achievement in American secondary schools. New York Teachers' College Press: New York.*

[24] *Newmann, F. M., Wehlage, G. G., & Lamborn. S. D. (1992) "The significance and sources of student engagement", in F. M. Newmann (ed.), Student engagement and achievement in American secondary schools. New York Teachers' College Press: New York.*

[25] *Anderson, O.D. (1977) 'The role of mathematics in today's society', International Journal of Mathematical Education in Science and Technology 8(4): 389-392*

[26] *Singh, Kusum, Granville, Monique & Dika, Sandra (2002) 'Mathematics and Science Achievement: Effects of Motivation, Interest, and Academic Engagement', The Journal of Educational Research 95(6): 323-332. pp. 323.*

5. Motivating students for mathematics learning

Motivating students is a complex task and now a commonly recognized problem[27,28] in HE institutions. While students have a role to play in motivating themselves, research has clearly shown[29,30] that how a teacher chooses to teach has an impact on students 'motivation, emotion and performance[31]'. While the common perception of motivation involves inspiring or stimulating another to complete a task, at SOWISO the key to motivation involves assessing student frustrations and providing a clear means to overcome them.

Some research has shown that the use of online digital platforms can be motivating for students who enabled to revise material repeatedly, outside the bounds of lecture hours; ask questions from their peers; or become exposed to frequent attempts at challenges[32]. However, teaching staff can do more to motivate their students and encourage them to engage more fully with digital platforms that can assist their learning. Anticipating points of frustration with the learning process and the teaching materials is an essential component of this. The following section considers various frustrations that students encounter, combined with different strategies teachers can follow to overcome these frustrations.

Problems: Students are frustrated by failure

Mathematics anxiety is 'prevalent' among the college students population (Betz 1978[33]) and there is a strong correlation between high anxiety and low motivation[34]. Students become frustrated when their attempts at problem-solving reach a barrier. While overcoming this barrier is the central component of learning, if students meet with frequent or long term failure, their motivation drops considerably.

Solution: Targeted feedback helps problem-solving

When students are lost or frustrated, a well designed learning system can help them find their way back. SOWISO uses targeted feedback to help students progress in problem-solving. This comes in the form of lines of text which pop up depending on the input of the student. When the answer is incorrect, the SOWISO platform will point out where and allow students additional

attempts. This immediate feedback helps students understand the way they're approaching problems and how to improve, allowing students to rapidly correct misconceptions and guide further study. In addition, each exercise has a number of on-demand hints that students can use before they even attempt to answer the question.

This decreases the frustration students feel when they do not know how to tackle a math problem, or when they cannot easily understand why their answer was deemed incorrect. Technology can tell students immediately how well they did on a specific task, and give them personalized tips on how to continue.

Problem: Students don't receive insights into their learning progress

A sense of progress is essential for motivation and engagement. Students need to feel as if they are advancing through material and that their learning and knowledge are improving week by week. However, charting one's own progress can be difficult, particularly in HE institutions where summative testing sometimes occurs as little as once per semester. Students are not able to gauge their progress or their achievements, ultimately demotivating them.

Solution: Milestone-based learning and trackable progress

Mastery-based learning is a set of group-based, individualized, teaching and learning strategies based on the premise that students will achieve a high level of understanding in a given domain if they are given enough time. On SOWISO, mastering a subject means the student was able to finish a number of exercises without mistakes, regardless of the amount of attempts it took a student to reach this level. This way, students can work on material as long as they need to, which provides a number of benefits. A simple colour system of red, orange and green shows students which topics they have mastered, providing them with an overview of past progress and future goals.

Using short (around 5 questions) formative quizzes on a weekly basis can also help students. If a teacher gathers a handful of exercises from the practice material which are representative of the topics and difficulty of the final exam, and puts them into a test (including randomized variables of course), students can use the outcomes of these weekly formative tests to not only see what topics they still need to work on, but also plan accordingly. This motivates students to study/do homework, and gives teachers insights into the progress of their students.

Problem: Students don't feel that they can study at their own pace

A great advantage of digital learning platforms is the ability for students to learn at their own individual pace. This approach provides many advantages[35] such as learning concepts slowly or quickly depending on preference and learning style, and the ability to re-cover material when necessary. This is particularly important in subjects such as maths where many students feel under confident in their abilities. On the other end of the spectrum, students who feel they already have an understanding of various foundational concepts can sometimes be frustrated by 'going slow' and needing to repeat topics they already understand in order to progress through the curriculum. Adaptive learning also allows these students to begin from their actual starting point rather than being forced to work through content they already understand.

Solution: Use adaptivity to create personalized learning journeys

Adaptivity allows students to walk their own learning path, perfectly tailored to their needs. Adaptive learning systems work by analyzing the performance of the students, and subsequently choose to make the material harder or easier. It adapts the path the students takes through the content so that it fits their level. Adaptive education technology does what a good teacher would do if there was enough time to give every student the personal care they deserve.

The primary focus of adaptivity is improving how a student experiences the flow of exercises. This is done by adjusting the pace and difficulty of the exercises students see on their screen. The first step in giving students fitting exercises is mapping their level of understanding. There are different ways of doing so, but due to the variety in student backgrounds and usages of the platform, SOWISO uses diagnostic tests. Students will be asked to complete these short tests when a new chapter is started. The algorithm can decide whether or not certain topics have to be reviewed by the student or if they are already mastered. A student can then continue to work on the topics which still need attention.

Instead of a fixed set of exercises, the adaptive platform will select the next exercise based on whether or not the previous exercise was answered correctly. The length of the set is therefore still unknown when a student starts practising.

Like a traditional solution, exercises get more difficult as the practice session goes on. Using an adaptive algorithm, the rate at which exercises will get more difficult is higher than the rate at which exercises get easier. This allows bright students to finish a chapter more quickly. Students for which the pace is too high can take a small step back without having to go back to the start. This is especially helpful for classrooms with a diverse body of students. Teachers can also opt for optional diagnostic tests at the beginning of a chapter. The diagnostic test uses the most difficult exercises of each chapter, so if students are able to pass the test, they

have proven their mastery over the subject. This means that advanced students can effectively skip entire topics based on their test performance.

Problem: Students are not motivated to regularly practice what they have learned

Students are frequently not motivated to regularly practice what they have learned in lectures and seminars. Unlike a school environment, where information is often spoon-fed to students in a supportive environment, university often leaves students to figure out their own learning style, including the frequency with which they practice new concepts.

While in school this issue would be addressed through homework, in the context of a large, weekly mathematics course at university, homework is rarely a feasible option for teachers. Students lack both an incentive to practice frequently, and a forfeit if they fail to practice.

Solution: Use achievement badges to incentivize students

Gamification is a new trend[36] in digital education which uses attributes traditionally used in games such as point scoring, winning badges, achieving challenges and competitive scoreboards to encourage learning by making it more fun and engaging. Gamification is a great way to engage students in daily learning as well as improving outcomes[37].

SOWISO uses light gamification in the form of skill-based achievement badges to motivate and encourage students as well as allowing them insight into their progress. Challenges become harder as a student progresses, introducing a sense of growth for students. In the future SOWISO will make more use of gamification aspects to continue to promote engagement.

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[27] Baillie, Caroline & Fitzgerald, Geraldine (2000) 'Motivation and attrition in engineering students', European Journal of Engineering Education 25(2): 145-155.

[28] Pintrich P.R., Zusho A. (2002) "Student Motivation and Self-Regulated Learning in the College Classroom", in Smart J.C., Tierney W.G. (eds) Higher Education: Handbook of Theory and Research. Higher Education: Handbook of Theory and Research, vol 17 Springer: Dordrecht.

[29] Deci, E. L., & Ryan, R. M. (1991) "A motivational approach to self: Integration in personality", in R. Dienstbier (Ed.), Nebraska Symposium on Motivation: Vol. 38. Perspectives on motivation (pp. 237-288). Lincoln: University of Nebraska Press.

[30] Reeve, J. (1996) Motivating others: Nurturing inner motivational resources. Needham Heights, MA: Allyn & Bacon.

[31] Reeve, Johnmarshall, Bolt, Elizabeth & Cai, Yi (1999) 'Autonomy-Supportive Teachers: How they teach and motivate students', Journal of Educational Psychology 91(3): 537-548.

[32] *Morrison, Karena & Camargo-Borges, Celiane (2016) "Reimagining the Purpose of Schools and Educational Organisations", in Montgomery, Anthony, Kehoe, Ian (Eds.) The Opportunities and Challenges of Using Digital Learning Environments in Educational Organizations pp 161-172. Springer: New York.*

[33] *Betz, N. (1978) 'Prevalence, distribution, and correlates of math anxiety in college students', Journal of Counseling Psychology 25(5): 441-48.*

[34] *Zakaria, Effandi & Norazah Mohd Nordin Eurasia Journal of Mathematics, Science & Technology Education, 2008, 4(1), 27-30 Copyright © 2008 by Moment E-ISSN: 1305-8223 The Effects of Mathematics Anxiety on Matriculation Students as Related to Motivation and Achievement*

[35] *Dawson, R., Georgiadou, E., Linecar, P., Ross, M., & Staples, G., (2006) "Learning and Teaching Issues in Software Quality", Proceedings of the 11th International Conference on Software Process Improvement - Research into Education and Training, April, Southampton, The British Computer Society, pp. 139-150.*

6. Conclusion

Learning can be a difficult and challenging task. This is particularly true for students who are required to study maths in order to progress in another chosen subject. For such students, making learning relevant, meaningful and accessible is a crucial part of improving engagement. While student engagement is strongly impacted by teaching activities including classroom time and face-to-face contact, digital learning platforms can also have a part to play in engaging and incentivising students in learning.

While digital learning platforms are sometimes dismissed as merely an attempt to make learning 'fun' for students (rather than more effective or helpful), the crucial task that these platforms can perform is to anticipate and consider frustrations that students experience and provide opportunities to overcome frustrations.

As discussed in this paper, frustrations appear at various stages in the learning process, including the initial stage of learning, practicing concepts and when considering their progress. While digital learning platforms cannot completely automate the learning/teaching process, they can make use of tools like milestone goals, achievement badges and targeted feedback. In addition to helping students overcome their frustrations, this approach allows students to feel more in control of their learning journey. In large classrooms this is particularly important as teaching staff do not always have the resources to provide individualized help.

SOWISO is focused on making mathematics learning for non-maths students as clear and easy as possible. This involves considering which hurdles students and teachers face and how best to overcome them with the assistance of digital technologies. We have achieved significant successes with our approach. To learn more about SOWISO, visit www.sowiso.nl/en/

7. About SOWISO

SOWISO is an interactive and personalized mathematics learning platform, designed for students uncomfortable with the subject. The platform guides students along an individual learning path, giving hints, identifying weaknesses and explaining complex concepts. SOWISO can be used by individuals or institutions as a support system for large classes of diverse learners.

SOWISO uses a variety of techniques to leverage mathematics teaching for a broad range of students. Several features make learning attractive and enjoyable for students and flexible and easy to integrate for teachers.

Personalized learning

The SOWISO platform analyses student work and highlights mistakes and weaknesses. The platform provides hints specific to a student's learning journey, meaning individual students get individual attention no matter how large a class is.

Adaptivity

Students can find their own path through maths content thanks to adaptive algorithms. The SOWISO platform constantly analyzes student comprehension and provides appropriate exercises. Testing provides insights into which topics have been mastered and which need more attention.

Learning Analytics

SOWISO provides extensive analytic data for every course. Teachers can gain valuable insights into how well students are learning, as well as diving deeper into subchapters, topics or even individual exercise attempts. Teachers can get information about individual students or the student body at large, allowing teachers to identify issues with content or individuals having trouble.

Automated testing

The SOWISO platform is capable of implementing randomized tests for students, checking and grading answers so that teachers do not have to undertake extra work. Numerous testing types are supported including diagnostic, formative or summative testing. Teachers can change the window of availability, number of attempts or a minimum passing grade among other adaptations.

Gamification

Teachers can activite gamification elements on the SOWISO platform, allowing students to gain achievement badges for important and informal tasks. Gamification further engages students and improves learning outcomes overall.

Integration

SOWISO can be fully integrated into an LMS like Moodle or Canvas or can be used in a cloud-based form using access codes and hyperlinks. Full integration allows for grades to be pushed back into an LMS gradebook, reducing administrative work for teachers

If your are interested in working with SOWISO, please contact us at:

info@sowiso.nl

Exercise

Students get detailed hints and feedback on every answer attempt

Feedback

Calculate the derivative $h'(x)$ of $h(x) = 7 \cdot \sin(8 \cdot x)$.

$h'(x) = 7 \cdot \cos(8 \cdot x)$ — No, you may have forgotten to multiply by $g'(x)$, in which $g(x) = 8 \cdot x$.

$h'(x) = 7 \cdot \cos(56 \cdot x)$ — The chain rule indicates that the derivative contains $f'(g(x))$, in which $f(x) = 7 \cdot \sin(x)$ and $g(x) = 8 \cdot x$. Thus, the the argument of the cosine must be equal to $g(x)$. This is not the case in your answer.

$h'(x) = 56 \cdot \cos(8 \cdot x)$ — Great job

Theory

We provide explanations with interactive elements

Trigonometry: Angles with sine, cosine and tangent

Angles in radians

So far we have expressed angles in degrees, but in mathematics angles are often expressed in radians. To introduce radians we will use a circle with radius 1. We call this the unit circle.

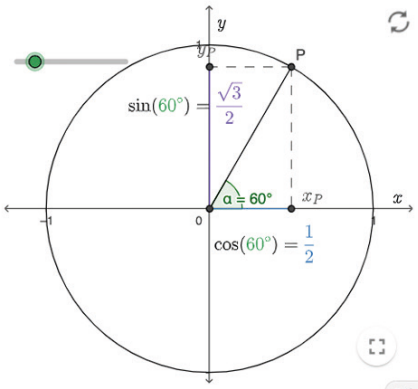
Unit circle

The **unit circle** is a circle with origin center $[0, 0]$ and radius 1.

The point $P = [x_P, y_P]$ starts at $[1, 0]$ and moves counterclockwise across the unit circle. The **angle of rotation** is called α.

Therefore $\sin(\alpha) = y_P$ and $\cos(\alpha) = x_P$.

In this way we can also define angles greater than $90°$ degrees with the sine and the cosine.



8. Bibliography

Anderson, O.D. (1977) 'The role of mathematics in today's society', International Journal of Mathematical Education in Science and Technology 8(4): 389-392

Attard, Catherine (2012) 'Engagement with mathematics: What does it mean and what does it look like?' Australian Primary Mathematics Classroom 17(1): 1-5.

Baillie, Caroline & Fitzgerald, Geraldine (2000) 'Motivation and attrition in engineering students', European Journal of Engineering Education 25(2): 145-155.

Bamforth, S., Robinson, C.L., Croft, T. & Crawford, A. (2007) 'Retention and progression of engineering students with diverse mathematical backgrounds', Teaching Mathematics and its applications, 26(4): 156-166.

Betz, N. (1978) 'Prevalence, distribution, and correlates of math anxiety in college students', Journal of Counseling Psychology 25(5): 441-48.

Bialek, W. & Botstein, D. (2004) 'Introductory science and mathematics education for 21st-century biologists', Science, 303(5659): 788-790.

Broadbridge. P. & Henderson, S. (2008) 'Mathematics Education for 21st Century Engineering Students: Final Report', Australian Mathematical Sciences Institute & Carrick Institute. Strawberry Hllls, NSW: Carrick Institute.

Brull, Stacey & Finlayson, Susan (2016) 'Importance of Gamification in Increasing Learning', The Journal of Continuing Education in Nursing 47(8): 372-375.

Cazzola, M. (2008), "Problem-Based Learning and Mathematics: Possible Synergical Actions", in L. Gomez Chova, D. M. Belenguer, and I. Candel Torres (eds.), ICERI2008 Proceeding, IATED (International Association of Technology, Education and Development), Valencia, Spain, 2008.

Croft, A.C., Harrison, M.C. & Robinson, C.L. (2009) 'Recruitment and retention of students - an integrated and holistic vision of mathematics support', International Journal of Mathematical Education in Science and Technology, 40(1): 109-125.

Dawson, R., Georgiadou, E., Linecar, P., Ross, M., & Staples, G., (2006) "Learning and Teaching Issues in Software Quality", Proceedings of the 11th International Conference on Software Process Improvement - Research into Education and Training, April, Southampton, The British Computer Society, pp. 139-150.

Deci, E. L., & Ryan, R. M. (1991) "A motivational approach to self: Integration in personality", in R. Dienstbier (Ed.), Nebraska Symposium on Motivation: Vol. 38. Perspectives on motivation (pp. 237-288). Lincoln: University of Nebraska Press.

Educational Horizons (2011) 'Flipping the classroom', Educational Horizons 90(1): 5-7.

Garrison, D. R. & T. Anderson (2003) E-learning in the 21st Century: A framework for research and practice. London: Routledge/Falmer.

Heck, A. (2017) 'Using SOWISO to realize interactive mathematical documents for learning, practising, and assessing mathematics', MSOR Connections 15(2).

Heck, A. & Brouwer, N. (2015) "Digital assessment-driven examples-based mathematics for computer science students", in N.Amado & Carreira (eds.) Proceedings of the 12th International Conference on Technology in Mathematics Teaching. University of Algarve, Portugal, pp.403-411.

Kapp, Karl M. (2013) The Gamification of Learning and Instruction Fieldbook: Ideas into Practice. John Wiley & Sons: Hoboken, New Jersey.

Lazakidou, Georgia & Retalis, Symeon (2010) 'Using computer supported collaborative learning strategies for helping students acquire self-regulated problem-solving skills in mathematics', Computers & Education 54: 3-13.

Llobregat-Gomez et al. (2015) cite a study which yielded an attendance rate of 95% with a 90% success rate for students, a number previously unheard of in the course.

Llorens, M., Nevin, E. & Mageen, E. et al. (2014) 'Online resource platform for mathematics education', in Proceedings 44th Annual Frontiers in Education (FIE) Conference (22-25 October) Madrid: 1865-1872.

London Mathematical Society (1995) Tackling the mathematics problem. London: London Mathematical Society. Available at: http://mei.org.uk/files/pdf/Tackling_the_Mathematics_Problem.pdf

Loughlin, Wendy A., Watters, Dianne J., Brown, Christopher L. & Johnston, Peter R. (2015) 'Snapshot of mathematical background demographics of a broad cohort or first year chemistry science students', International Journal of Innovation in Science and Mathematics Education 23(1): 21-36.

Morrison, Karena & Camargo-Borges, Celiane (2016) "Reimagining the Purpose of Schools and Educational Organisations", in Montgomery, Anthony, Kehoe, Ian (Eds.) The Opportunities and Challenges of Using Digital Learning Environments in Educational Organizations pp 161-172. Springer: New York.

Ndloui, Mdutshekelwa C. & Mostert, I. (2018) 'Teacher Perceptions of Moodle and Throughput in a Blended Learning Programme for In-Service Secondary School Mathematics Teachers', Africa Education Review, 15(2): 131-151.

Newmann, F. M., Wehlage, G. G., & Lamborn. S. D. (1992) "The significance and sources of student engagement", in F. M. Newmann (ed.), Student engagement and achievement in American secondary schools. New York Teachers' College Press: New York.

Pintrich P.R., Zusho A. (2002) "Student Motivation and Self-Regulated Learning in the College Classroom", in Smart J.C., Tierney W.G. (eds) Higher Education: Handbook of Theory and Research. Higher Education: Handbook of Theory and Research, vol 17 Springer: Dordrecht.

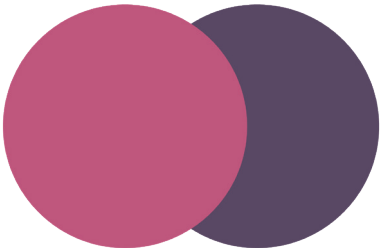
Reeve, J. (1996) Motivating others: Nurturing inner motivational resources. Needham Heights, MA: Allyn & Bacon.

Reeve, Johnmarshall, Bolt, Elizabeth & Cai, Yi (1999) 'Autonomy-Supportive Teachers: How they teach and motivate students', Journal of Educational Psychology 91(3): 537-548.

Şendağ, S. & Odabaşı, H. F., 'Effects of an online problem based learning course on content knowledge acquisition and critical thinking skills', Computers & Education 53(1): 132-142.

Singh, Kusum, Granville, Monique & Dika, Sandra (2002) 'Mathematics and Science Achievement: Effects of Motivation, Interest, and Academic Engagement', The Journal of Educational Research 95(6): 323-332. pp.324.

Zakaria, Effandi & Norazah Mohd Nordin (2008) 'The Effects of Mathematics Anxiety on Matriculation Students as Related to Motivation and Achievement', Eurasia Journal of Mathematics, Science & Technology Education 4(1): 27-30.



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